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Structural Control Using Inverse H_2 Optimal Theory

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The piezoelectric transducers embedded in structures offer distributed sensor and actuator architectures for structural control, a process that involves health monitoring and control of local structural degrees of freedom. The design steps demand sensor and actuator management schemes so that the response of a structural component in question is monitored and then tailored to the design requirements. Kalman's inverse theory is used to determine sensor and sensor–actuator combinations for optimal observers and controllers. The Riccati equations for each combination are derived to formulate the linear quadratic Gaussian theory for a reconfigurable architecture. An oscillator model with two inputs is considered to illustrate the underlying structural control principles.

Nomenclature

$\boldsymbol{A}$	=	state vector
b	=	input matrix
C	=	matrix relating measurement and state vectors
c	=	damping
D	=	control influence matrix
$d(t)$	=	exogeneous input
$\boldsymbol{e}$	=	error vector
H	=	positive semidefinite symmetric matrix
$h(s)$	=	transfer function
I	=	identity matrix
J	=	performance index (cost function)
j	=	$\sqrt{-1}$
K	=	stiffness matrix
k, k_c	=	stabilizing controller
kC	=	control gain matrix
k_f	=	Kalman filter gain matrix
k_1, k_2	=	stiffnesses
L	=	Luenberger observer gain matrix
ℓ_a	=	number of actuators
ℓ_s	=	number of sensors
M	=	mass matrix
m_1, m_2	=	masses
N	=	damping matrix
P, S	=	positive definite symmetric matrix
P_f	=	filter Riccati matrix
p	=	number of performance variables
Q	=	noise matrix
R^n	=	n-dimensional Cartesian space
s	=	Laplace variable
$\boldsymbol{s}(t)$	=	acceleration vector
$\dot{\boldsymbol{s}}(t)$	=	displacement vector
$\ddot{\boldsymbol{s}}(t)$	=	velocity vector
t	=	time
$\boldsymbol{u}(t)$	=	control vector
V, W	=	noise intensity matrices
$v(t)$	=	measurement noise
$w(t)$	=	system noise
X	=	covariance matrix of $\boldsymbol{x}(t)$
$\boldsymbol{x}(t)$	=	state estimation vector
$\hat{\boldsymbol{x}}(t)$	=	state vector
$\boldsymbol{Y}_i$	=	ith basis vector
$\boldsymbol{y}(t)$	=	performance vector
$y_i(t)$	=	ith performance variable
$z(t)$	=	measurement vector
$\Delta A, \Delta b, \Delta C$	=	variations in A, b, C due to uncertainties
ε	=	expected value operator
ξ	=	modal coordinate
ω	=	eigenvalue
$'$	=	transpose

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I. Introduction

THE capabilities of piezoelectric transducers embedded in structures introduced various active structures in the past. Intelligent,[1] adaptive,[2] biologically inspired structures,[3] etc., are some examples of active structures defined in the literature. The finite element method can be used to model structures embedded with piezoelectric materials. In this method, discrete piezoelectric transducers are assumed to be located at element nodes, whereas distributed piezoelectric materials are used in deriving proper element characteristic matrices and vectors by incorporating their mechanical, electrical and coupling effects.[4] Thus, the method permits the modeling and analysis of the resulting structure as a multi-degree-of-freedom system. Although a common control objective in most of these structures is to tailor the dynamic response of structural components considered in a finite element (FE) model, neither the structural degrees of freedom (SDOF) that demand performance enhancement nor a controller that supplements the performance are known a priori. The problem becomes complex if a healthy structure becomes damaged in the presence of modeling and environmental uncertainties. In this case, the SDOF could refer the damaged structural components with mass and stiffness variations in the FE models.[5,6] The SDOF could also refer the residual modes or the truncated structural modes,[7] affected by an exogenous input, sensor noise, spillover, etc. The piezoelectric transducers available as both sensors and actuators are widely seen as an alternative to reconfigure a sensor and actuator combination that detects and modifies the response of defective SDOF.

In linear quadratic Gaussian (LQG) theory (see Refs. 8 and 9), the separation principle accepts a state feedback H_2 optimal controller and integrates it with an observer that monitors the SDOF in question. Although LQG formulation is suitable for structural health monitoring and control, the controller and observer in a fixed sensor architecture (FSA) is required to handle damping and stiffness modifications within a structural component and its various SDOF resulting from the internal and external nodes of p and h versions.[10] Also direct optimal design assumes a known performance vector, noise sources, etc., and seeks a controller that is robust to the specified modeling and measurement uncertainties. However, in health monitoring and control, none of them are known. Finally, if the structural component with either a sensor or an actuator is damaged, then a procedure to disengage these components from a fixed sensor and actuator (FSA) architecture is necessary for the safe operation of structure.

Clearly, structural control (SC) in the presence of mixed uncertainties (damage, residual modes, exogenous input, noise, etc.,) is a process that involves health monitoring, detection of local SDOF demanding performance enhancement, and a control input at a healthy structural component so that the operation of structure in an uncertain environment is safe. In this paper, LQG formulation in a reconfigurable sensor architecture (RSA) is introduced. It works in the framework of an FSA but has an ability to engage or disengage a certain combination of sensors that are not favorable to a defective structural component. Kalman's inverse theory (see Ref. 11), in this context, offers various sensor and actuator combinations and derives optimal controllers and observers as required in RSA.

It is known that a controller satisfying the Kalman's optimality criterion in state and out-put feedback setting also guarantee infinite gain margin (see Ref. 12), a fundamental property the separation principle in LQG formulation uses to integrate the controller and observer gains.[13,14] In Sec. II, a brief background on inverse theory is presented. In Sec. III, the SC problem is defined. If a controller for a sensor–actuator combination or an observer for a sensor combination is found optimal, then a procedure to compute the Riccati equations (REs) is presented in Sec. IV. For each of these controller and observer RE pairs, in Sec. V the LQG formulation for RSA is presented. Given LQG formulation, SC schemes in RSA framework are discussed in Sec. VI. The attributes of RSA for local structural mode control using a fixed proportional plus derivative (PD) controller is presented in Sec. VII. In Sec. VIII, an oscillator model is used to illustrate the SC principles. Conclusions and remarks are presented in Sec. IX.

II. Background

Consider a vector $\boldsymbol{s}(t) \in R^{n/2}$ with the displacement and rotational SDOF at FE nodes. Let the model for a healthy structure be

$$M\ddot{\boldsymbol{s}}(t) + N\dot{\boldsymbol{s}}(t) + K\boldsymbol{s}(t) = D\boldsymbol{u}(t) \tag{1}$$

The velocity and acceleration vectors are represented by $\dot{\boldsymbol{s}}(t)$ and $\ddot{\boldsymbol{s}}(t)$, respectively. Let $\boldsymbol{u}(t) = R^{\ell_a}$ be the number of transducers embedded in structure as actuators and let D be the control influence matrix that usually depends on the transducer location, type, etc. For actuator management schemes, $\boldsymbol{u}(t)$ is assumed to be a scalar. Let M, N, and K represent the mass, damping, and stiffness matrices of appropriate dimensions. The first-order state-space system required to address direct and inverse optimal control theory is

$$\underbrace{\begin{bmatrix}\dot{\boldsymbol{s}}(t)\\ \ddot{\boldsymbol{s}}(t)\end{bmatrix}}_{\dot{x}(t)} = \underbrace{\begin{bmatrix}0 & I\\ -M^{-1}K & -M^{-1}N\end{bmatrix}}_{A}\underbrace{\begin{bmatrix}\boldsymbol{s}(t)\\ \dot{\boldsymbol{s}}(t)\end{bmatrix}}_{x(t)} + \underbrace{\begin{bmatrix}0\\ M^{-1}D\end{bmatrix}}_{b}\boldsymbol{u}(t) \tag{2}$$

Let $\boldsymbol{z}(t) \in R^{\ell_s}$ be the transducers embedded in a structure as sensors, and let the measurement vector be $\boldsymbol{z}(t) = \boldsymbol{C}\boldsymbol{x}(t)$. In direct H_2 optimal control, the objective is to seek a control law

$$\boldsymbol{u}(t) = -kC\boldsymbol{x}(t) \tag{3}$$

such that the known performance variables $\boldsymbol{y}_1(t), \ldots, \boldsymbol{y}_p(t)$ stacked in a column vector $\boldsymbol{y}(t) = H\boldsymbol{x}(t)$ is optimal with respect to the linear quadratic cost function,

$$J = \int_0^\infty \{x'(t)H'Hx(t) + u'(t)u(t)\}\,\mathrm{d}t, \qquad \boldsymbol{x}(0) \neq 0 \tag{4}$$

Note that the performance vector is known as a linear combination of the state variables. In addition, the signals from all of the actuators $u_1, \ldots, u_{\ell_a}$ and sensors $z_1, \ldots, z_{\ell_s}$ are simultaneously processed to seek a controller $k^{\ell_a \times \ell_s}$ that minimizes J. The controller needs to satisfy the following optimality constraints:

$$P = P' \tag{5}$$

$$(A - bkC)'P + P(A - bkC) = -(H'H + C'k'kC) \tag{6}$$

$$(A - bkC)'S + S(A - bkC) = -X \tag{7}$$

$$b'PSC'(CSC')^{-1} = k \tag{8}$$

Given H and k, the algebraic RE [(ARE), Eq. (6)] solves for a positive definite symmetric matrix P. Given X, the covariance matrix for $\boldsymbol{x}(t)$, the Lyapunov equation (7) solves for a positive definite symmetric matrix S. Both P and S define the optimal controller in Eq. (8). Various parameter optimization techniques have been proposed to solve for k and its constraints given in Eqs. (5–8).

Inverse theory, on the other hand, assumes a stabilizing controller $k^{1\times \ell_s}$ and determines a class of performance vectors $\boldsymbol{y}(t)$ such that the optimality criterion is met. In state feedback setting,[15] inverse theory assumes $\ell_s = n$ and determines whether k is optimal for all possible H and J. The criterion is

$$|1 + k(j\omega I - A)^{-1}b|^2 > 1, \qquad \forall \omega > 0 \tag{9}$$

All of these controllers also satisfy the following algebraic optimality criterion:

$$P = P', \qquad b'P = k$$

$$(A - bk)'P + P(A - bk) - k'k = -H'H \tag{10}$$

Clearly, inverse theory seeks the unknown performance vectors $\boldsymbol{y}(t) = H\boldsymbol{x}(t)$ such that the controller k for a controllable system minimizes J. A simple extension of inverse theory for an output feedback controller ρkC that seeks infinite gain margin is given by[11]

$$\rho = \sup_{\omega > 0} \frac{-2\cos[\angle h(j\omega)]}{|h(j\omega)|} \tag{11}$$

where $h(s) = kC(sI_n - \boldsymbol{A})^{-1}b$. The algebraic optimality criterion modifies to

$$P = P', \qquad \rho kC (= k_c) = b'P$$

$$(\boldsymbol{A} - bk_c)'P + P(\boldsymbol{A} - bk_c) = -(H'H + k_c'k_c) \tag{12}$$

Note that the covariance matrix X in this case is $H'H + k_c'k_c$ and S is indeed P. The duality of controller RE is the observer RE, given as

$$L' = CP_L, \qquad \boldsymbol{A}P_L + P_L\boldsymbol{A}' - P_LC'CP_L + Q = 0 \tag{13}$$

The observer RE (13) is guaranteed to exist if the conditions 1) $(\boldsymbol{A} - LC)$ is stable, 2) $L'C' > 0$, and 3) $Y_LC' = 0$ hold for a symmetric Y_L (Ref. 14). An equivalent statement of these conditions is as follows.[12] If $(C, \boldsymbol{A})$ is observable and $(\boldsymbol{A}, \sqrt{Q})$ where $Q \geq 0$ is reachable, then the error system

$$\dot{\boldsymbol{e}}(t) = (\boldsymbol{A} - LC)\boldsymbol{e}, \qquad \boldsymbol{e} = \boldsymbol{x}(t) - \hat{\boldsymbol{x}}(t) \tag{14}$$

is asymptotically stable. For a given control law $\boldsymbol{u}(t)$, the estimate of the states $\hat{\boldsymbol{x}}(t)$ from an observer is inferred from

$$\frac{\mathrm{d}\hat{\boldsymbol{x}}(t)}{\mathrm{d}t} = \boldsymbol{A}\hat{\boldsymbol{x}}(t) + b\boldsymbol{u}(t) + LC[\boldsymbol{x}(t) - \hat{\boldsymbol{x}}(t)] \tag{15}$$

In the context of a deterministic FE model, Luenberger observer gains L do not necessarily guarantee the RE. Suppose L satisfies observer RE; note that Q has no implication as in controller RE, wherein we gave the relation $Q = H'H$. LQG theory, however, interprets Q as noise sources in the system equations and handles structured control problems defined next.

III. SC Problem Formulation

Consider a FE model with modeling and environmental uncertainties,

$$\begin{bmatrix}\dot{\boldsymbol{x}}(t)\\ \dot{\xi}(t)\end{bmatrix}=\begin{bmatrix}\boldsymbol{A}+\Delta\boldsymbol{A} & u\\ u & u\end{bmatrix}\begin{bmatrix}\boldsymbol{x}(t)\\ \xi(t)\end{bmatrix}+\begin{bmatrix}b+\Delta b\\ u\end{bmatrix}\boldsymbol{u}(t)+\begin{bmatrix}e\\ e\end{bmatrix}d(t)+Gw(t) \tag{16}$$

$$z(t)=(C+\Delta C)\boldsymbol{x}(t)+v(t) \tag{17}$$

where the scalars $d(t)$, $w(t)$, and $v(t)$ represent the exogenous input, system noise, and measurement noise, respectively. The additional SDOF introduced due to the modified FEs in p and h versions are denoted by ξ. The structural modes associated with ξ are referred to as residual modes that were ignored in the controller and observer design procedures. The coupling terms in the matrix elements with letters u and e refer to the contributions of the unmodeled dynamics and exogenous input, respectively. All of the properties, such as controllability, stability, optimality, etc., under which a controller and an observer were designed, may become invalid in the presence of combined uncertainty $\{\Delta A, \Delta b, \Delta C, d, w, v\}$. A risk-mitigating algorithm should be able to detect and repair $\boldsymbol{y}(t)$ for the safe operation of structures. Thus SC in RSA becomes important. A specific problem considered in this paper is to determine various sensor and actuator combinations so that the controllers and observers in each combination are available for SC in active structures.[1–3]

A. Open-Loop Structural Health Monitoring and Control

A healthy structure with active sensors is assumed for analysis. Among the given sensors $z_1, z_2, \ldots, z_{\ell_s}$, it is possible to reconfigure 1, or 2, ..., or ℓ_s sensors at a time. The total number of sensor combinations available for RSA are the sum of ${}^{\ell_s}C_1, {}^{\ell_s}C_2, \ldots, {}^{\ell_s}C_{\ell_s}$, respectively. Out of these possibilities, the sensor combinations that satisfy the conditions in Eq. (13) are of particular interest for observer REs. Under ideal operating conditions without modeling and environmental uncertainty sources, the sensor combinations that guarantee observer REs offer stable observable systems. However, in the presence of uncertainties, the responses from these observers are expected to isolate the presence of a defective structural component. The goal is to develop a switching criterion that reconfigures a sensor combination for which a controller in output feedback setting ensures safe operation of the structure. Various switching schemes for adaptive control are proposed in the literature.[16] In structures, however, it is important to maintain the periodic response of individual structural models in case any one of them is mistuned.[17] If a structural model is mistuned, any localization effects introduced can be reduced or eliminated by controlling the location of the coupling constraint. A switching criterion can be used to engage sensors whose responses from observers are periodic and disengage all sensors whose responses are either aperiodic (not periodic), unbounded or oscillatory.

B. Closed-Loop Structural Health Monitoring and Control

The problem in closed loop is similar to that of the open-loop structural health monitoring and control problem. The structure, however, is now active by a controller in FSA. The observer outputs in the active structure are usually bounded unless the structure is perturbed by the presence of mixed uncertainties $\{\Delta A, \Delta b, \Delta C, d, w, v\}$. If a structural component embedded with a sensor or actuator is found defective, then its response (the response of SDOF associated with a defective component) is expected to become aperiodic. The problem in closed-loop structural health monitoring and control is to detect the SDOF whose responses are aperiodic and use an RSA such that the structure has periodic oscillations. The total number of actuator combinations available for each sensor combination defined in Sec. III.A are the sum of ${}^{\ell_a}C_1, {}^{\ell_a}C_2, \ldots, {}^{\ell_a}C_{\ell_a}$, respectively. As in sensor case, actuator combinations satisfying the conditions in Eq. (12) for a controller RE are of particular interest. The SC problem and switching criterion for periodic regulation are similar to that of the open-loop case. In addition, because the structure is now active, the necessary requirement is to detect and disengage a sensor or actuator, whose responses are aperiodic.

IV. Inverse H_2 Theory: Implications

Deterministic linear quadratic optimal control (or H_2 optimal control) has been an important design tool in control of structures. In particular, output feedback controllers subject to the constraints in Eqs. (5–8) require parameter optimization.[18] Although piezoelectric transducers are available all over the surface of a structure, it is often difficult to derive a FE model with a measurement vector for state feedback control problem. Thus, SC in static output feedback setting[19,20] is of considerable interest in a control system design.

Inverse theory suggests that any controller designed for a minimal-phase loop transfer function $h(s)$ with pole-zero excess less than three is H_2 optimal. The controller further inherits guaranteed stability margins found in a state feedback regulator, a property used in the LQG formulation. The dual REs in LQG formulation are derived in detail. Consider a stabilizing controller $k_c=\rho kC$ for a controllable and an observable pair $(\boldsymbol{A}, b)$ and $(C, \boldsymbol{A})$, respectively. 1) Compute the basis $\boldsymbol{Y}_i \in R^{n\times n}$ for $Yb=0$, where

$$Y=\sum_{i=1}^{\delta}\alpha_i\boldsymbol{Y}_i$$

is symmetric for all nonzero vectors $\boldsymbol{\alpha}=[\boldsymbol{\alpha}_1,\ldots,\boldsymbol{\alpha}_\delta]$. 2) Use the components of $\boldsymbol{\alpha}$ such that $P=\bar{P}(\rho)+Y$ is positive definite for some $\boldsymbol{\alpha}^*$, where $\bar{P}(\rho)=\rho C'k'(kCb)^{-1}kC$. 3) Substituting P and rewriting the ARE, we get unknown performance vectors $\boldsymbol{y}(t)=H\boldsymbol{x}(t)$ and symmetric matrices $H'H$ as

$$H'H=-\boldsymbol{A}'\left[\bar{P}+\sum_{i=1}^{\delta}\boldsymbol{\alpha}_i\boldsymbol{Y}_i\right]-\left[\bar{P}+\sum_{i=1}^{\delta}\boldsymbol{\alpha}_i\boldsymbol{Y}_i\right]\boldsymbol{A}+k_c'k_c \tag{18}$$

For every $\boldsymbol{\alpha}^*$, there exists several performance vectors for which the controller ρkC is LQ optimal and P is PD. The number of performance variables in each vector $\boldsymbol{y}(t)$ depends on the dimension of H, namely, $p\le n$.

V. LQG Theory: Implications

Although inverse theory directly incorporates sensor measurements and offers stability margins similar to that of a state feedback regulator, traditional measurement feedback systems develop controllers that are robust to the noise characteristics. LQG theory assumes stochastic FE models driven by the white noise $w(t)$ and $v(t)$ and determines a full state feedback controller k linear to the estimated states $\hat{\boldsymbol{x}}$. Because k_c in output feedback setting inherits the properties of k, LQG formulation for a RSA is proposed. In this connection, the symmetric matrices in observer RE are interpreted as follows: Let ε be the expected value operator. Define $\varepsilon\{ww'\}=W\ge 0$ and $\varepsilon\{vv'\}=V>0$. The Kalman filter gain

$$k_f=P_fC'V^{-1} \tag{19}$$

and $P_f\ge 0$ satisfies the filter RE

$$P_f\boldsymbol{A}'+\boldsymbol{A}P_f-P_fC'V^{-1}CP_f+GWG'=0 \tag{20}$$

Note that the observer and filter REs have same interpretation, but in the former case, V and W are not identified [Eq. (13)]. Suppose $V=I_{\ell_s\times\ell_s}$ and $W=I_{n\times n}$; the Q becomes GG'. As in controller RE, the inverse approach for observer RE is useful to isolate the unknown input noise vector $\boldsymbol{w}(t)$ through G. The integrated controller and observer scheme becomes

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix}\boldsymbol{x}(t)\\ \hat{\boldsymbol{x}}(t)\end{bmatrix}=\begin{bmatrix}\boldsymbol{A} & -bkC\\ k_fC & A-k_fC-bkC\end{bmatrix}\begin{bmatrix}\boldsymbol{x}(t)\\ \hat{\boldsymbol{x}}(t)\end{bmatrix}+\begin{bmatrix}G\boldsymbol{w}(t)\\ k_fv(t)\end{bmatrix} \tag{21}$$

The augmented system matrix in Eq. (21) is of considerable interest in deriving the separation principle. Introducing the parametric uncertainties and performing the row and column operations, the perturbed eigenvalues of the integrated system matrix are inferred as[21]

$$\det\begin{bmatrix} sI-(A-k_fC)-E_0 & 0 \\ -k_f(C+\Delta C) & sI-(A-bkC)-E_c \end{bmatrix}=0 \quad (22)$$

where $E_0=\Delta A-k_f\Delta C$ and $E_c=\Delta A-\Delta bk$ are the modeling uncertainties affecting the controller and observer poles. If ΔC, $\Delta b=0$, and if the model preserves controllability and observability properties with E_0, $E_c=0$, then the convergence of Kalman filter may hold even if the integrated system is unstable. However, in the presence of generic uncertainties $\{\Delta A, \Delta b, \Delta C, d, w, v\}$, filter and controller gains designed for a specific sensor and actuator configuration may not preserve various assumptions such as controllability, stability, optimality, etc. In this case, LQG formulation for RSA seeks another sensor and actuator combination for which the Kalman filter converges; however, this aspect of our investigation is beyond the scope of this paper.

VI. SC Using RSA

The sources of modeling uncertainties are evident during the time the FE nodes are picked. The internal and external nodes in p and h versions of FE modeling, in theory, can make n denumerably large. However, for all practical design and analysis purposes, the model is truncated with finite DOF. The SDOF ignored to have a finite dimensional model contributes to unstructured uncertainty. Likewise, the growth of structural defect from a tiny crack to large damage introduce stochastic and deterministic parameter variations. In this section, SC using RSA is discussed to monitor, isolate, and mitigate defects resulting from such uncertainties. Consider an FSA and a set of sensors C and the elements of k_c populated as shown by $\times$:

$$\begin{array}{rcccc} & s-1 & s-2 & s-3 & s-4 \\ k_c=a-1 & \times & \times & \times & \times \\ a-2 & \times & \times & \times & \times \end{array}$$

Controller in this case refers to two actuators and two sensors that are active. If, for some reason, sensors s-1 and s-3 are required to be deactivated, then the controller for RSA takes the form

$$\begin{array}{rcccc} & s-1 & s-2 & s-3 & s-4 \\ k_c=a-1 & 0 & \times & 0 & \times \\ a-2 & 0 & \times & 0 & \times \end{array}$$

Further, deactivating an actuator located at a defective structural component, the actuator management is inferred as

$$\begin{array}{rcccc} & s-1 & s-2 & s-3 & s-4 \\ k_c=a-1 & 0 & \times & 0 & \times \\ a-2 & 0 & 0 & 0 & 0 \end{array}$$

The 2-actuator and 4-sensor case is generalized for a finite number of sensors ℓ_s and actuators ℓ_a embedded in a structure. Existence of observer REs for some pattern in each sensor combination $C\in R^{\ell_s\times n}$ is presented. In deriving a controller, a single actuator is assumed to be active. That is, $\boldsymbol{b}\in R^n$ is a column vector. Deterministic and probabilistic structural control schemes are presented. In the deterministic case, integrated SC scheme results in

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix}\boldsymbol{x}(t)\\ \hat{\boldsymbol{x}}(t)\end{bmatrix}=\begin{bmatrix}A & -\boldsymbol{b}k_c\\ LC & A-LC-\boldsymbol{b}k_c\end{bmatrix}\begin{bmatrix}\boldsymbol{x}(t)\\ \hat{\boldsymbol{x}}(t)\end{bmatrix} \quad (23)$$

The RSA in this setting is useful to handle disturbance $d(t)$ and parameter variations $(\Delta \boldsymbol{A}, \Delta \boldsymbol{b})$ defined for a large damage at the local coordinates of a structure. To study noise characteristics and other probabilistic structural parameter variations,[22] an observer RE in LQG setting is necessary. In this case, the formulation for RSA is

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix}\boldsymbol{x}(t)\\ \hat{\boldsymbol{x}}(t)\end{bmatrix}=\begin{bmatrix}A & -\boldsymbol{b}k_c\\ k_fC & A-k_fC-\boldsymbol{b}k_c\end{bmatrix}\begin{bmatrix}\boldsymbol{x}(t)\\ \hat{\boldsymbol{x}}(t)\end{bmatrix}+\begin{bmatrix}Gw(t)\\ k_fv(t)\end{bmatrix} \quad (24)$$

Note that the control law, $\boldsymbol{u}(t)=-k_c\boldsymbol{x}(t)$, in the deterministic case maintains infinite gain margin. However, in the LQG setting, the control law modifies to

$$\boldsymbol{u}(t)=-k_c\hat{\boldsymbol{x}}(t) \quad (25)$$

In summary, each controller ρkC in RSA minimizes the expected value of J in Eq. (4), such that the performance vector $\boldsymbol{y}(t)=\boldsymbol{H}\boldsymbol{x}(t)$ for a stochastic or a deterministic FE model in Eqs. (16) and (17) is optimal. Inverse theory computes H and balances the controller RE as in Eq. (18) if a positive definite P at α^* is known. Likewise, G is balanced using the observer RE defined for a known sensor noise characteristics V. The attributes of LQG and H_2 formulation in RSA is useful to control the independent SDOF, explained in the sequel.

VII. Local Structural Mode Control

Local structural mode control in a FSA is similar to that of the independent modal space control that attempts to tailor the response of all modes considered in a FE model using the position and velocity feedback. In a natural coordinate system,[19,20] it is shown that the PD controller of the form

$$\rho kC=\begin{bmatrix}k_p^1 & 0 & \ldots & k_d^1 & 0 & \ldots & \text{(for mode 1)}\end{bmatrix}$$
$$=\begin{bmatrix}0 & k_p^2 & \ldots & 0 & k_d^2 & \ldots & \text{(for mode 2)}\end{bmatrix}$$

guarantees an RE of order two for every mode. In local structural model control, an original coordinate system is considered. A procedure to detect and control a particular structural mode shown in Fig. 1 is presented. A fixed-order PD controller for an RSA is

$$u(t)=-k_px_i(t)-k_d\dot{x}_i(t), \qquad i\in[1,2,\ldots,\ell_s]$$
$$=-\underbrace{\begin{bmatrix}0 & 0 & \underbrace{k_p}_{i} & 0 & \ldots & 0 & 0 & \underbrace{k_d}_{n+i} & \ldots\end{bmatrix}}_{k_c}x(t)$$

Note that the sensors could be either $\boldsymbol{x}_i(t)$ for a PD controller or $\{x_i, \dot{x}_i\}$ for a proportional controller. Figure 1 presents two collocated sensor and actuator pairs, one located at a healthy component

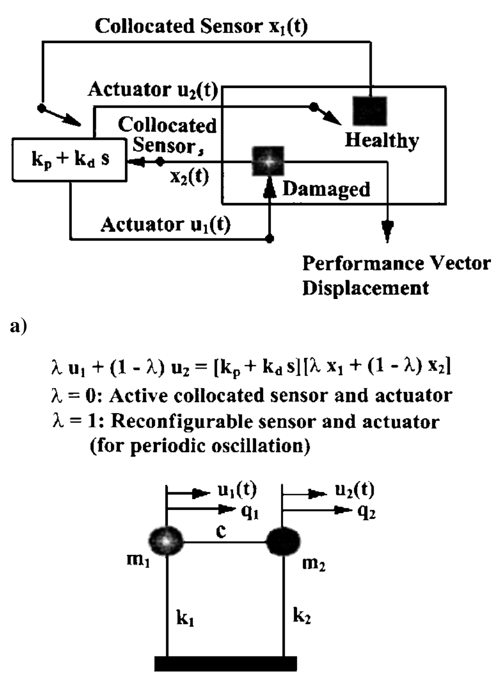


Fig. 1 a) Piezoelectric transducers at healthy and damaged structural components and b) reduced-order oscillator model illustrating structural control principles.

and another at a damaged component. The oscillator model is used to explain this behavior of such structures. Suppose the collocated pair $\{u_1, x_1, \dot{x}_1\}$ is active; then the control law is given by

$$\boldsymbol{u}_1(t) = -k_p \boldsymbol{x}_1(t) - k_d \dot{\boldsymbol{x}}_1(t)$$

If an observer for this active structure exhibits $\hat{\boldsymbol{x}}_1(t)$ aperiodic and $\hat{\boldsymbol{x}}_2(t)$ periodic, then $\boldsymbol{u}_1(t)$ is naturally aperiodic. Thus, it is necessary to reconfigure a sensor whose response is periodic such that the actuator input linear to these sensors is also periodic. Two possibilities exist:

$$\boldsymbol{u}_1(t) = -k_p \boldsymbol{x}_2(t) - k_d \dot{\boldsymbol{x}}_2(t)$$

$$\boldsymbol{u}_2(t) = -k_p \boldsymbol{x}_2(t) - k_d \dot{\boldsymbol{x}}_2(t)$$

Furthermore in RSA, if both $\hat{x}_1$ and $\hat{x}_2$ are periodic, then the model as a whole maintains periodic regulation. In this case, a simple switching rule to manage sensors and actuators is necessary. The actuator management may be given as

$$\boldsymbol{u}(t) = \lambda \boldsymbol{u}_1(t) + (1-\lambda)\boldsymbol{u}_2(t), \qquad \lambda = 0 \text{ or } 1$$

Similarly, the sensor management is

$$\boldsymbol{u}(t) = -[k_p + k_d s][\lambda \boldsymbol{x}_1(t) + (1-\lambda)x_2(t)], \qquad \lambda = 0 \text{ or } 1$$

where $\lambda = 0$ refers the collocated pair $\{u_1, x_1, \dot{x}_1\}$ that is active and $\lambda = 1$ reconfigures the pair $\{u_2, x_2, \dot{x}_2\}$ for periodic regulation of the oscillator.

VIII. Oscillator Example

In this section, the oscillator in Fig. 1 is modeled using a collocated sensor and actuator pairs at masses m_1 and m_2, $\{u_1, x_1, \dot{x}_1\}$ and $\{u_2, x_2, \dot{x}_2\}$ (Ref. 23),

$$\begin{bmatrix} \dot{\boldsymbol{x}}_1(t) \\ \dot{\boldsymbol{x}}_2(t) \\ \ddot{\boldsymbol{x}}_1(t) \\ \ddot{\boldsymbol{x}}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & -0.2 & 0.2 \\ 0 & -2 & 0.2 & -0.2 \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_1(t) \\ \boldsymbol{x}_2(t) \\ \dot{\boldsymbol{x}}_1(t) \\ \dot{\boldsymbol{x}}_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \boldsymbol{u}_1(t) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \boldsymbol{u}_2(t)$$

Suppose the measurement vector $\boldsymbol{z}(t)$ in an FSA is $\dot{x}_1, \dot{x}_2, x_1$, and x_2, then the state and output feedback controllers are given by

$$k_c = \begin{bmatrix} k_p^1 & k_p^2 & k_d^1 & k_d^2 \end{bmatrix} \boldsymbol{x}(t), \qquad n = 4$$

$$k_c = \begin{bmatrix} k_p^1 & k_p^2 & \ldots & k_d^1 & k_d^2 & \cdots \end{bmatrix} \boldsymbol{x}(t), \qquad n > 4$$

For the oscillator model with $n = 4$ and a fixed PD controller with $k_p = k_p^i$ and $k_d = k_d^i$, the two sensor combinations considered are,

$$k_c = [k_p \quad 0 \quad k_d \quad 0] \qquad (\text{for sensor } x_1, \dot{x}_1)$$

$$= [0 \quad k_p \quad 0 \quad k_d] \qquad (\text{for sensor } x_2, \dot{x}_2)$$

Of the four actuator management schemes, the two schemes satisfying the conditions for the existence of controller REs are

$$\boldsymbol{u}_1(t) = [k_d \quad 0 \quad k_p \quad 0]\boldsymbol{x}(t)$$

or

$$\boldsymbol{u}_2(t) = [0 \quad k_d \quad 0 \quad k_p]\boldsymbol{x}(t)$$

The infinite gain margin controller satisfying the LQ optimality condition in Eq. (11) is $k_p = 3$ and $k_d = 30$. The nonsingularity constraint in Eq. (11) for the two actuator schemes are shown in Fig. 2.

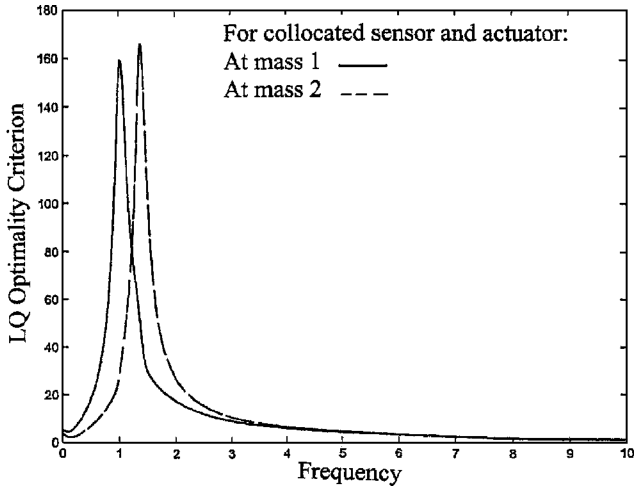


Fig. 2 Condition in Eq. (11) for a PD controller, 3(1 + 10 s).

Note that the constraint is derived for frequency in the interval [0,10]. The controller RE for the sensor and actuator pair $\{u_1, x_1, \dot{x}_1\}$ is

$$Y = \begin{bmatrix} \alpha_1 & \alpha_2 & 0 & \alpha_3 \\ \alpha_2 & \alpha_4 & 0 & \alpha_5 \\ 0 & 0 & 0 & 0 \\ \alpha_3 & \alpha_5 & 0 & \alpha_6 \end{bmatrix}$$

One of the combinations of $\boldsymbol{\alpha}$ that makes P positive definite is $\boldsymbol{\alpha}^* = [1, 0, 0, 1, 0, 1]$. The unknown performance vector $\boldsymbol{y}(t)$ for $\boldsymbol{\alpha}^*$ is computed as

$$\boldsymbol{y}(t) = \begin{bmatrix} 3.9604 & -2.6141 & -16.1787 & -25.2482 \\ -0.0000 & -0.6982 & -0.6805 & 0.5084 \\ 0.9533 & 0.0059 & 0.0712 & 0.1033 \\ -0.0053 & -0.6062 & 0.4519 & -0.2276 \end{bmatrix} \boldsymbol{x}(t)$$

Similarly the controller RE for the sensor and actuator pair $\{u_2, x_2, \dot{x}_2\}$ is

$$Y = \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 & 0 \\ \alpha_2 & \alpha_4 & \alpha_5 & 0 \\ \alpha_3 & \alpha_5 & \alpha_6 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

and $\boldsymbol{y}(t)$ for this combination is given by

$$\boldsymbol{y}(t) = \begin{bmatrix} 0 & 0 & 0 & 30.5049 \\ 0.3053 & -1.8493 & 0.1970 & 0 \\ -0.0042 & 0.0665 & 0.6305 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \boldsymbol{x}(t)$$

The observer RE for the sensors $\{x_1, \dot{x}_1\}$ is

$$Y_L = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \beta_1 & 0 & \beta_2 \\ 0 & 0 & 0 & 0 \\ 0 & \beta_2 & 0 & \beta_3 \end{bmatrix}$$

For $\boldsymbol{\alpha}^* = [0.1, 0, 0.1]$, the noise sources G in the FE model is given by

$$G = \begin{bmatrix} 0.1838 & 0.0919 & -0.3552 & 0.1868 \\ -0.1093 & 0.1978 & 0.1040 & 0.2079 \\ -0.0913 & 0.2018 & -0.0960 & -0.1920 \\ 0.1655 & 0.0827 & 0.0882 & -0.0358 \end{bmatrix}$$

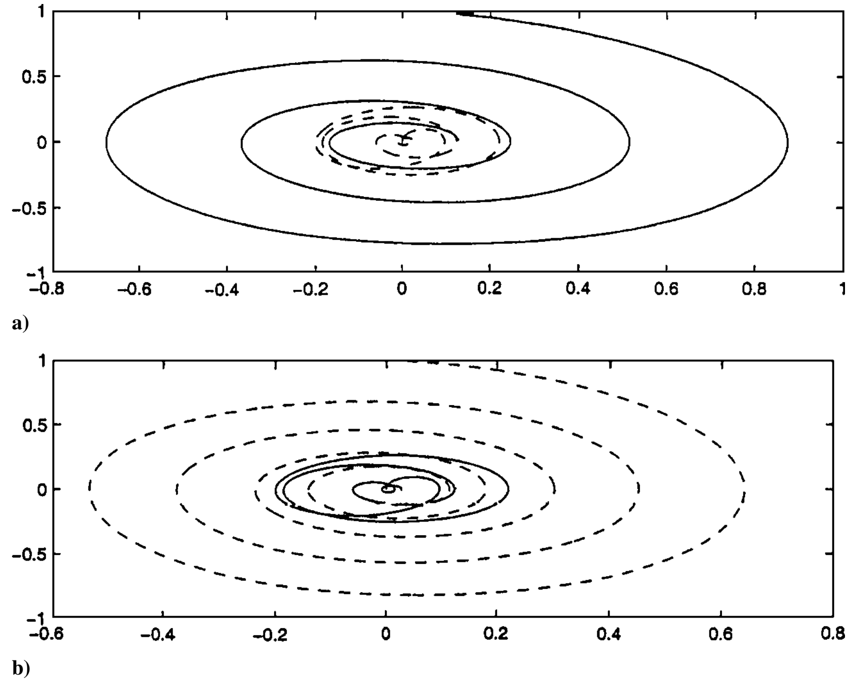


Fig. 3 Open-loop structural health monitoring; a) $\hat{x}_1$ vs $\hat{\dot{x}}_1$ and b) $\hat{x}_2$ vs $\hat{\dot{x}}_2$: Luenberger observer response for an impulse applied at ——, mass 1 or – – –, mass 2.

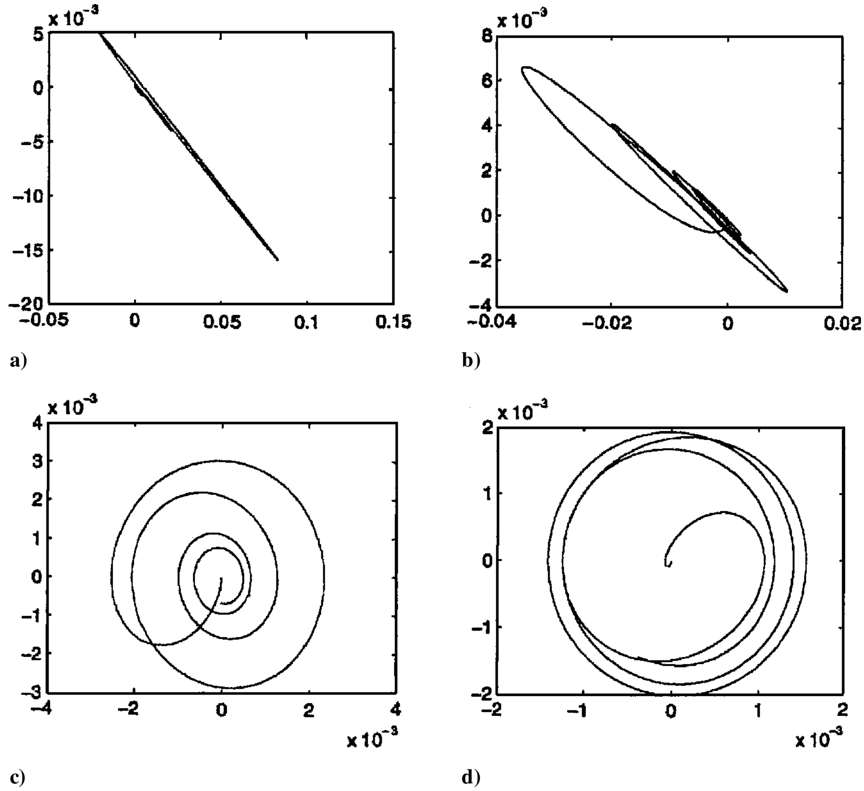
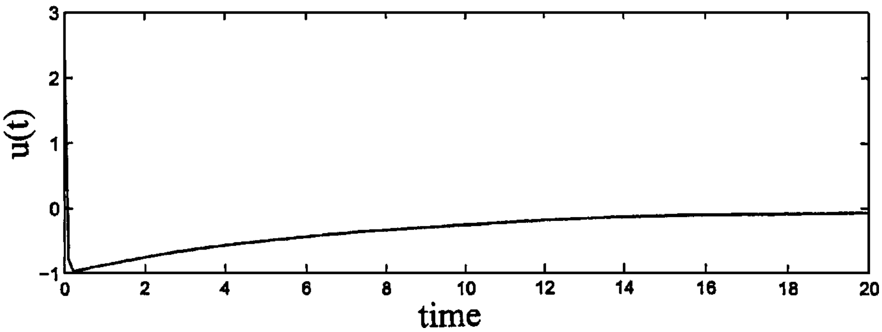


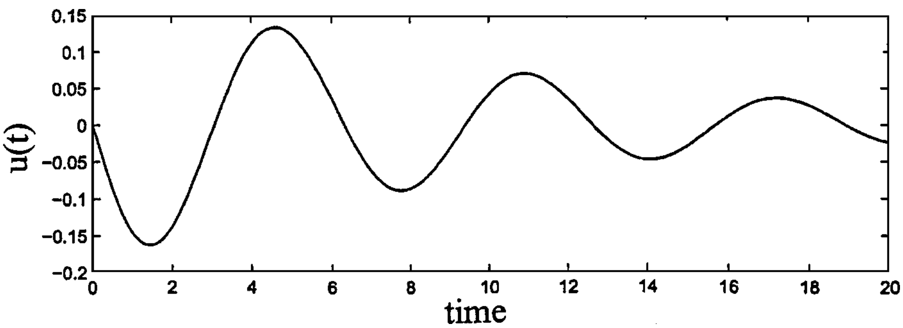
Fig. 4 Health monitoring in active structures; Luenberger observer response for initial conditions at a) mass 1, $\hat{x}_1$ vs $\hat{\dot{x}}_1$; b) mass 2, $\hat{x}_1$ vs $\hat{\dot{x}}_1$; or c) mass 1, $\hat{x}_2$ vs $\hat{\dot{x}}_2$; and mass 2, $\hat{x}_2$ vs $\hat{\dot{x}}_2$.

The controllers $k_c = b'P$ and observers $L = C P_L$ or $k_f = P_f C' V^{-1}$ for deterministic and stochastic models satisfy the algebraic optimality criteria. Note that in the deterministic case, the Leunberger observer L is not required to be optimal.

Next, the sensor combination $\{x_1, \dot{x}_1\}$ and sensor–actuator combinations $\{u_1, x_1, \dot{x}_1\}$ and $\{u_2, x_2, \dot{x}_2\}$, respectively, are used to illustrate the SC principles. The goal is to preserve periodic regulation if an observer response in the presence of modeling and environmental uncertainties is found aperiodic. In open loop, the observer response in Fig. 3 for initial conditions and an impulse suggest both x_1 and x_2 are periodic. However, if the structure is active with a collocated sensor and actuator pair $\{u_1, x_1, \dot{x}_1\}$, the observer response in Fig. 4 suggests that x_1 is aperiodic for initial conditions at mass 2, whereas x_2 is periodic for initial conditions at both mass 1 and 2, respectively (Figs. 4c and 4d). It is necessary to disengage sensors $\{x_1, \dot{x}_1\}$ and engage $\{x_2, \dot{x}_2\}$ by an actuator $\boldsymbol{u}_1(t)$ or $\boldsymbol{u}_2(t)$ so that a control input linear to these measurements is periodic as shown in Fig 5. Thus, the switching criterion employed in this context is the



a) PD controller for sensors at mass 1 (aperiodic stabilizing control input)



b) PD controller for sensors at mass 2 (periodic stabilizing control input)

Fig. 5 Switching criterion using principle of linearity and superposition.

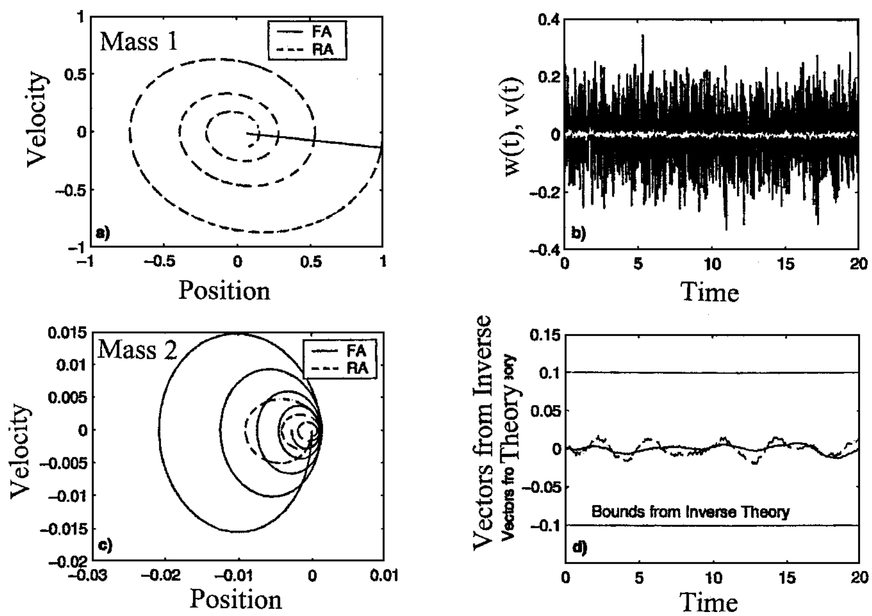


Fig. 6 Intelligent system via Kalman's inverse theory; restoration of periodic oscillator response using reconfigurable architecture for deterministic exogenous inputs at mass 1 or for probabilistic noise sources at either mass 1 or mass 2.

principles of linearity and superposition. Note that the optimality criterion excludes the choice $\{u_1, x_2, \dot{x}_2\}$ because it fails to satisfy the condition $kCb > 0$. Thus, the actuator u_2 for sensors $\{x_2, \dot{x}_2\}$ is reconfigured. By this choice, the responses of x_1 and x_2 becomes periodic, as shown in Figs. 6a and 6c. Furthermore, LQG formulation provides the response of oscillator (Fig. 6d) driven by the noise (Fig. 6c). The covariance matrix resulting from the observer RE further suggest the bounds within which the oscillator response is confined. Characterization of G used to balance the observer RE requires careful attention. Suppose $\boldsymbol{w}(t)$ is scalar, then G is spanned by the vectors $[\mathbf{0};\ \mathbf{0};\ \boldsymbol{g}_1;\ \mathbf{0}]$ and $[\mathbf{0};\ \mathbf{0};\ \mathbf{0};\ \boldsymbol{g}_2]$, so that the oscillator model with white noise $\boldsymbol{w}(t)$ may be written as

$$\ddot{x}_1 + 2\zeta(\dot{x}_1 - \dot{x}_2) + x_1 = \boldsymbol{u}_1(t) + \boldsymbol{g}_1\boldsymbol{w}(t) \tag{26}$$

$$\ddot{x}_2 + 2(\zeta/\mu)(\dot{x}_1 - \dot{x}_2) + (\kappa/\mu)x_2 = \boldsymbol{u}_2(t) + \boldsymbol{g}_2\boldsymbol{w}(t) \tag{27}$$

Because we specifiy G and seek k_f,balancing observer RE using P and a meaningful G becomes a complex problem.

IX. Conclusions

This paper develops a procedure to use the piezoelectric transducers embedded with structures in a reconfigurable sensor architecture. It works in the frame work of an FSA, but has an ability to engage or disengage a sensor or actuator so that the remaining sensors and actuators function within the framework of H_2 and LQG optimal theory. Kalman's inverse theory is used to design various controllers and observers for a reconfigurable architecture. The idea is to use these controllers and observers and control structures with mixed uncertainties, which include damage, noise, disturbance inputs, etc. In proposing this methodology, it is observed that balancing the observer RE using symmetric matrices requires further attention to understand probabilistic structural models. However, the general idea of reconfigurable control is used to address the periodic regulation of local structural modes, for which the controller and observer combination is used in a deterministic sense.

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